

CP violation for $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ in QCD factorization

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Abstract. In the QCD factorization (QCDF) approach we study the direct CP violation in $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ via the $\rho-\omega$ mixing mechanism. We find that the CP violation can be enhanced by double $\rho-\omega$ mixing when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance, and the maximum CP violation can reach 28%. We also compare the results from the naive factorization and the QCD factorization.

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1 Introduction

CP violation is an extensive research topics in recent years. In Standard Model (SM), CP violation is related to the weak complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2]. In the past few years more attention has been focused on the decays of B meson system both theoretically and experimentally. Recently, the large CP violation was found by the LHCb Collaboration in the three-body decay channels of $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$ and $B^\pm \rightarrow K^\pm\pi^+\pi^-$ [3,4]. Hence, the theoretical mechanism for the three or four-body decays become more and more interesting. In this paper, we focus on the interference from intermediate ρ and ω mesons in the four-body decay.

It is known that the naive factorization [5,6], the QCD factorization (QCDF) [7,8,9], the perturbative QCD (PQCD) [10,11,12], and the soft-collinear effective theory (SCET) [13,14] are the most extensive approaches for calculating the hadronic matrix elements. These factorization approaches present different methods for dealing with the hadronic matrix elements in the leading power of $1/m_b$ (m_b is the b-quark mass). Direct CP violation occurs through the interference of two amplitudes with different weak phases and strong phases. The weak phase difference is directly determined by the CKM matrix elements, while the strong phase is usually difficult to control. However, and not well

determined from a theoretical approach. The B meson decay amplitude involves the hadronic matrix elements which computation is not trivial. Different methods may present different strong phases. Meanwhile, we can also obtain a large strong phase difference by some phenomenological mechanism. $\rho-\omega$ mixing has been used for this purpose in the past few years [15,16,17,18,19,20,21,22,23,24,25]. In this paper, we will investigate the CP violation via double $\rho-\omega$ mixing in the QCDF approach.

In the QCDF approach, at the rest frame of the heavy B meson, B meson can decay into two light mesons with large momenta. In the heavy-quark limit, QCD corrections can be calculated for the non-leptonic two-body B -meson decays. The decay amplitude can be obtained at the next-to-leading power in α_s and the leading power in A_{QCD}/m_b . In the QCD factorization, there is cancellation of the scale and renormalization scheme dependence between the Wilson coefficients and the hadronic matrix elements. However, this does not happen in the naive factorization. The hadronic matrix elements can be expressed in terms of form factors and meson light-cone distribution amplitudes including strong interaction corrections.

The remainder of this paper is organized as follows. In Sec. 2 we present the form of the effective Hamiltonian. In Sec. 3 we give the calculating formalism of CP violation from $\rho-\omega$ mixing in $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. Input parameters are presented in Sec.4. We present the numerical results in Sec.5. Summary and discussion are included in Sec. 6.

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2 The effective hamiltonian

With the operator product expansion, the effective weak Hamiltonian can be written as [26]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{p=u,c} \sum_{q=d,s} V_{pb} V_{pq}^* (c_1 O_1^p + c_2 O_2^p + \sum_{i=3}^{10} c_i O_i + c_{7\gamma} O_{7\gamma} + c_{8g} O_{8g}) \right] + H.c., \quad (1)$$

where G_F represents the Fermi constant, c_i ($i = 1, \dots, 10, 7\gamma, 8g$) are the Wilson coefficients, V_{pb} , V_{pq} are the CKM matrix elements. The operators O_i have the following forms:

$$\begin{aligned} O_1^p &= \bar{p}\gamma_\mu(1-\gamma_5)b\bar{q}\gamma^\mu(1-\gamma_5)p, \\ O_2^p &= \bar{p}\alpha\gamma_\mu(1-\gamma_5)b_\beta\bar{q}\beta\gamma^\mu(1-\gamma_5)p_\alpha, \\ O_3 &= \bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q', \\ O_4 &= \bar{q}\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \\ O_5 &= \bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q', \\ O_6 &= \bar{q}\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'\beta\gamma^\mu(1+\gamma_5)q'_\alpha, \\ O_7 &= \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q', \\ O_8 &= \frac{3}{2}\bar{q}\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'\beta\gamma^\mu(1+\gamma_5)q'_\alpha, \\ O_9 &= \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q', \\ O_{10} &= \frac{3}{2}\bar{q}\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \\ O_{7\gamma} &= \frac{-e}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b, \\ O_{8g} &= \frac{-g_s}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b, \end{aligned} \quad (2)$$

where α and β are color indices, O_1^p and O_2^p are the tree operators, $O_3 - O_6$ are QCD penguin operators which are isosinglets, $O_7 - O_{10}$ arise from electroweak penguin operators which have both isospin 0 and 1 components. $O_{7\gamma}$ and O_{8g} are the electromagnetic and chromomagnetic dipole operators, $e_{q'}$ are the electric charges of the quarks and $q' = u, d, s, c, b$ is implied.

For the decay channel $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega)$, neglecting power corrections of order Λ_{QCD}/m_b , the transition matrix element of an operator O_i in the weak effective Hamiltonian is given by [8, 9]

$$\begin{aligned} \langle V_1 V_2 | O_i | \bar{B} \rangle &= \sum_j F_j^{B \rightarrow V_1}(m_{V_2}^2) \int_0^1 du T_{ij}^I(u) \Phi_{V_2}(u) \\ &+ (V_1 \leftrightarrow V_2) \\ &+ \int_0^1 d\xi du dv T_i^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{V_1}(v) \Phi_{V_2}(u) \end{aligned} \quad (3)$$

Here $F_j^{B \rightarrow V_{1,2}}(m_{V_{2,1}}^2)$ denotes $B \rightarrow V_{1,2}$ ($V_{1,2}$ represent ρ^0 and ω mesons) form factor, and $\Phi_V(u)$ is the light-cone distribution amplitude for the quark-antiquark Fock state of mesons ρ^0 and ω . $T_{ij}^I(u)$ and $T_i^{II}(\xi, u, v)$ are hard-scattering functions, which are perturbatively calculable.

The hard-scattering kernels and light-cone distribution amplitudes (LCDA) depend on the factorization scale and the renormalization scheme. $m_{V_{1,2}}$ denote the ρ^0 and ω masses, respectively.

We match the effective weak Hamiltonian onto a transition operator, the matrix element is given by ($\lambda_p^{(D)} = V_{pb} V_{pD}^*$ with $D = d$) [8, 9]

$$\langle V_1 V_2 | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = \sum_{p=u,c} \lambda_p^{(D)} \langle V_1 V_2 | \mathcal{T}_A^{p,h} + \mathcal{T}_B^{p,h} | \bar{B} \rangle. \quad (4)$$

where $\mathcal{T}_A^{p,h}$ denotes the contribution from vertex correction, penguin amplitude and spectator scattering in terms of the operators $a_i^{p,h}$, $\mathcal{T}_B^{p,h}$ refers to annihilation terms contribution by operators $b_i^{p,h}$. h is the helicity of the final state.

The flavor operators a_i^p are defined in [8, 9] as follows:

$$\begin{aligned} a_i^{p,h}(V_1 V_2) &= \left(c_i + \frac{c_{i\pm 1}}{N_c} \right) N_i^h(V_2) \\ &+ \frac{c_{i\pm 1}}{N_c} \frac{C_F \alpha_s}{4\pi} \left[V_i^h(V_2) + \frac{4\pi^2}{N_c} H_i^h(V_1 V_2) \right] \\ &+ P_i^{p,h}(V_2), \end{aligned} \quad (5)$$

where N_c is the number of colors, the upper (lower) signs apply when i is odd (even), and $C_F = \frac{N_c^2 - 1}{2N_c}$. It is understood that the superscript 'p' is to be omitted for $i = 1, 2$. The quantities $V_i^h(V_2)$ account for one-loop vertex corrections, $H_i^h(V_1 V_2)$ for hard spectator interactions, and $P_i^{p,h}(V_2)$ for penguin contractions. $N_i^h(V_2)$ is given by

$$N_i^h(V_2) = \begin{cases} 0; & i = 6, 8, \\ 1; & \text{all other cases.} \end{cases} \quad (6)$$

The coefficients of the flavor operators $\alpha_i^{p,h}$ can be expressed in terms of the coefficients $a_i^{p,h}$. We will present the form in the following section. Using the unitarity relation

$$\lambda_u^{(D)} + \lambda_c^{(D)} + \lambda_t^{(D)} = 0, \quad (7)$$

we can get

$$\begin{aligned} &\sum_{p=u,c} \lambda_p^{(D)} \mathcal{T}_A^{p,h} \\ &= \sum_{p=u,c} \lambda_p^{(D)} \left[\delta_{pu} \alpha_1(V_1 V_2) A([\bar{q}_s u][\bar{u} D]) \right. \\ &\quad + \delta_{pu} \alpha_2(V_1 V_2) A([\bar{q}_s D][\bar{u} u]) \left. \right] + \lambda_u^{(D)} \left[(\alpha_4^u(V_1 V_2) \right. \\ &\quad - \alpha_4^c(V_1 V_2)) \sum_q A([\bar{q}_s q][\bar{q} D]) + (\alpha_{4,\text{EW}}^u(V_1 V_2) \\ &\quad - \alpha_{4,\text{EW}}^c(V_1 V_2)) \sum_q \frac{3}{2} e_q A([\bar{q}_s q][\bar{q} D]) \left. \right] \end{aligned}$$

$$\begin{aligned}
& -\lambda_t^{(D)} \left[\alpha_3^c(V_1 V_2) \sum_q A([\bar{q}_s D][\bar{q} q]) \right. \\
& + \alpha_4^c(V_1 V_2) \sum_q A([\bar{q}_s q][\bar{q} D]) \\
& + \alpha_{3,\text{EW}}^c(V_1 V_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s D][\bar{q} q]) \\
& \left. + \alpha_{4,\text{EW}}^c(V_1 V_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s q][\bar{q} D]) \right], \quad (8)
\end{aligned}$$

where the sums extend over $q = u, d, s$, and \bar{q}_s denotes the spectator antiquark.

Next we need to change the annihilation part into the following form [8, 9]:

$$\begin{aligned}
\sum_{p=u,c} \lambda_p^{(D)} \mathcal{T}_B^{p,h} &= \sum_{p=u,c} \lambda_p^{(D)} \\
&\times \left[\delta_{pu} b_1(V_1 V_2) \sum_{q'} B([\bar{u} q'][\bar{q}' u][\bar{D} b]) \right. \\
&+ \delta_{pu} b_2(V_1 V_2) \sum_{q'} B([\bar{u} q'][\bar{q}' D][\bar{u} b]) \left. \right] \\
&- \lambda_t^{(D)} \left[b_3(V_1 V_2) \sum_{q,q'} B([\bar{q} q'][\bar{q}' D][\bar{q} b]) \right. \\
&+ b_4(V_1 V_2) \sum_{q,q'} B([\bar{q} q'][\bar{q}' q][\bar{D} b]) \\
&+ b_{3,\text{EW}}(V_1 V_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q} q'][\bar{q}' D][\bar{q} b]) \\
&\left. + b_{4,\text{EW}}(V_1 V_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q} q'][\bar{q}' q][\bar{D} b]) \right], \quad (9)
\end{aligned}$$

where $b_i^{p,h}$, $b_{i,\text{EW}}^{p,h}$ and B will be given in the following section.

3 CP violation in

$$B^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$$

3.1 Formalism

The $B \rightarrow V_1(\epsilon_1, P_1)V_2(\epsilon_2, P_2)$ ($\epsilon_1(P_1)$ and $\epsilon_2(P_2)$ are the polarization vectors (momenta) of V_1 and V_2 , respectively) decay rate is written as

$$\Gamma = \frac{G_F^2 P_c}{64\pi m_B^2} \sum_{\sigma} A^{(\sigma)+} A^{(\sigma)}, \quad (10)$$

where P_c refers to the c.m. momentum. $A^{(\sigma)}$ is the helicity amplitude for each helicity of the final state. The decay

amplitude, A , can be decomposed into three components H_0 , H_+ , H_- according to the helicity of the final state. With the helicity summation, we can get

$$\sum_{\sigma} A^{(\sigma)+} A^{(\sigma)} = |H_0|^2 + |H_+|^2 + |H_-|^2. \quad (11)$$

In the vector meson dominance model [27], the photon propagator is dressed by coupling to vector mesons. Based on the same mechanism, $\rho - \omega$ mixing was proposed [28, 29]. The formalism for CP violation in the decay of a bottom hadron, \bar{B} , will be reviewed in the following. The amplitude for $\bar{B} \rightarrow V\pi^+\pi^-$, A , can be written as

$$A = \langle \pi^+\pi^- V | H^T | \bar{B} \rangle + \langle \pi^+\pi^- V | H^P | \bar{B} \rangle, \quad (12)$$

where H^T and H^P are the Hamiltonians for the tree and penguin operators, respectively. We define the relative magnitude and phases between these two contributions as follows:

$$A = \langle \pi^+\pi^- V | H^T | \bar{B} \rangle [1 + r e^{i\delta} e^{i\phi}], \quad (13)$$

where δ and ϕ are strong and weak phase differences, respectively. The weak phase difference ϕ arises from the appropriate combination of the CKM matrix elements: $\phi = \arg[(V_{tb}V_{td}^*)/(V_{ub}V_{ud}^*)]$. The parameter r is the absolute value of the ratio of tree and penguin amplitudes,

$$r = \left| \frac{\langle \pi^+\pi^- V | H^P | \bar{B} \rangle}{\langle \pi^+\pi^- V | H^T | \bar{B} \rangle} \right|. \quad (14)$$

The amplitude for $B \rightarrow \bar{V}\pi^+\pi^-$ is

$$\bar{A} = \langle \pi^+\pi^- \bar{V} | H^T | B \rangle + \langle \pi^+\pi^- \bar{V} | H^P | B \rangle. \quad (15)$$

Then, the CP violating asymmetry, A_{CP} , can be written as

$$\begin{aligned}
A_{CP} &= \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \\
&= \frac{-2(T_0^2 r_0 \sin \delta_0 + T_+^2 r_+ \sin \delta_+ + T_-^2 r_- \sin \delta_-) \sin \phi}{\sum_{i=0+-} T_i^2 (1 + r_i^2 + 2r_i \cos \delta_i \cos \phi)}, \quad (16)
\end{aligned}$$

where

$$|A|^2 = \sum_{\sigma} A^{(\sigma)+} A^{(\sigma)} = |H_0|^2 + |H_+|^2 + |H_-|^2 \quad (17)$$

and $T_i (i = 0, +, -)$ represent the tree-level helicity amplitudes. We can see explicitly from Eq. (16) that both weak and strong phase differences are needed to produce CP violation. $\rho - \omega$ mixing has the dual advantages that the strong phase difference is large and well known [15, 16]. In this scenario one has

$$\begin{aligned}
\langle \pi^+\pi^-\pi^+\pi^- | H^T | \bar{B} \rangle &= \frac{2g_\rho^2}{s_\rho^2 s_\omega} \tilde{H}_{\rho\omega}(t_\omega + t_\omega^a) \\
&+ \frac{g_\rho^2}{s_\rho^2} (t_\rho + t_\rho^a), \quad (18)
\end{aligned}$$

$$\langle \pi^+\pi^-\pi^+\pi^- | H^P | \bar{B} \rangle = \frac{2g_\rho^2}{s_\rho^2 s_\omega} \tilde{\Pi}_{\rho\omega}(p_\omega + p_\omega^a) + \frac{g_\rho^2}{s_\rho^2} (p_\rho + p_\rho^a), \quad (19)$$

where t_V ($V = \rho$ or ω) is the tree amplitude and p_V is the penguin amplitude for producing a vector meson, V . t_V^a ($V = \rho$ or ω) is the tree annihilation amplitude and p_V^a is the penguin annihilation amplitude. g_ρ is the coupling for $\rho^0 \rightarrow \pi^+\pi^-$, $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho - \omega$ mixing amplitude, and s_V is from the inverse propagator of the vector meson V ,

$$s_V = s - m_V^2 + im_V \Gamma_V, \quad (20)$$

with \sqrt{s} being the invariant mass of the $\pi^+\pi^-$ pair. The direct $\omega \rightarrow \pi^+\pi^-$ is effectively absorbed into $\tilde{\Pi}_{\rho\omega}$, leading to the explicit s dependence of $\tilde{\Pi}_{\rho\omega}$ [30,31]. Making the expansion $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$, the $\rho - \omega$ mixing parameters were determined in the fit of Gardner and O'Connell [32]: $\text{Re}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300 \text{ MeV}^2$, $\text{Im}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300 \text{ MeV}^2$, and $\tilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04$. In practice, the effect of the derivative term is negligible. From Eqs. (16)(18), one has

$$r e^{i\delta} e^{i\phi} = \frac{2\tilde{\Pi}_\omega(p_\omega + p_\omega^a) + s_\omega(p_\rho + p_\rho^a)}{2\tilde{\Pi}_{\rho\omega}(t_\omega + t_\omega^a) + s_\omega(t_\rho + t_\rho^a)}. \quad (21)$$

Defining

$$\frac{t_\omega + t_\omega^a}{t_\rho + t_\rho^a} = \alpha e^{i\delta_\alpha}, \quad (22)$$

$$\frac{p_\rho + p_\rho^a}{p_\omega + p_\omega^a} = \beta e^{i\delta_\beta}, \quad (23)$$

$$\frac{p_\omega + p_\omega^a}{t_\rho + t_\rho^a} = r' e^{i(\delta_q + \phi)}, \quad (24)$$

where δ_α , δ_β , and δ_q are strong phases, one finds the following expression from Eq. (21):

$$r e^{i\delta} = r' e^{i\delta_q} \frac{2\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta} s_\omega}{s_\omega + 2\tilde{\Pi}_{\rho\omega} \alpha e^{i\delta_\alpha}}. \quad (25)$$

$\alpha e^{i\delta_\alpha}$, $\beta e^{i\delta_\beta}$, and $r e^{i\delta}$ will be calculated in the QCD factorization approach in the next section. With Eq. (25), we can obtain $r \sin \delta$ and $r \cos \delta$. In order to get the CP violating asymmetry, A_{CP} , in Eq. (16), $\sin \phi$ and $\cos \phi$ are needed. ϕ is determined by the CKM matrix elements. In the Wolfenstein parametrization [33,34], one has

$$\sin \phi = \frac{\eta}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}, \quad (26)$$

$$\cos \phi = \frac{\rho(1-\rho) - \eta^2}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}. \quad (27)$$

3.2 The calculation details

In the QCD factorization approach, α_i associated with the coefficients a_i can be written as follows (helicity indices are neglected)[8,9]:

$$\alpha_1 = a_1 \quad (28)$$

$$\alpha_2 = a_2 \quad (29)$$

$$\alpha_3^p = a_3^p + a_5^p \quad (30)$$

$$\alpha_{3,EW}^p = a_9^p + a_7^p \quad (31)$$

$$\alpha_4^p = a_4^p - r_\chi^{V_2} a_6^p \quad (32)$$

$$\alpha_{4,EW}^p = a_{10}^p - r_\chi^{V_2} a_8^p, \quad (33)$$

where we have used the notation

$$r_\chi^V \equiv \frac{2m_V}{m_b} \frac{f_V^\perp}{f_V}. \quad (34)$$

with f_V^\perp , f_V referring to the transverse decay constant and decay constant of the vector meson, respectively.

The flavor operators $a_i^{p,h}$ include short-distance non-factorizable corrections such as vertex corrections and hard spectator interactions. V_2 is the emitted meson and V_1 shares the same spectator quark with the B meson.

The vertex corrections are given by[8,9]:

$$V_i^0(V_2) = \begin{cases} \int_0^1 dx \Phi_{\parallel}^{V_2}(x) \left[12 \ln \frac{m_b}{\mu} - 18 + g(x) \right], & (i = 1-4, 9, 10) \\ \int_0^1 dx \Phi_{\parallel}^{V_2}(x) \left[-12 \ln \frac{m_b}{\mu} + 6 - g(1-x) \right], & (i = 5, 7) \\ \int_0^1 dx \Phi_{v_2}(x) \left[-6 + h(x) \right], & (i = 6, 8) \end{cases} \quad (35)$$

$$V_i^\pm(V_2) = \begin{cases} \int_0^1 dx \Phi_{\pm}^{V_2}(x) \left[12 \ln \frac{m_b}{\mu} - 18 + g_T(x) \right], & (i = 1-4, 9, 10) \\ \int_0^1 dx \Phi_{\mp}^{V_2}(x) \left[-12 \ln \frac{m_b}{\mu} + 6 - g_T(1-x) \right], & (i = 5, 7) \\ 0, & (i = 6, 8) \end{cases} \quad (36)$$

with

$$g(x) = 3 \left(\frac{1-2x}{1-x} \ln x - i\pi \right)$$

$$\begin{aligned}
& + \left[2\text{Li}_2(x) - \ln^2 x + \frac{2\ln x}{1-x} \right. \\
& \quad \left. - (3 + 2i\pi) \ln x - (x \leftrightarrow 1-x) \right], \\
h(x) &= 2\text{Li}_2(x) - \ln^2 x - (1 + 2i\pi) \ln x - (x \leftrightarrow 1-x), \\
g_T(x) &= g(x) + \frac{\ln x}{\bar{x}}, \tag{37}
\end{aligned}$$

where $\bar{x} = 1 - x$, Φ_{\parallel}^V is a twist-2 light-cone distribution amplitude of the meson V , Φ_{V_2} (for the longitudinal component) and Φ_{\pm} (for transverse components) are twist-3 ones.

Hard spectator terms

$H_i^h(V_1 V_2)$ arise from hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the \bar{B} meson. $H_i^0(V_1 V_2)$ have the expressions[8,9]:

$$\begin{aligned}
H_i^0(V_1 V_2) &= \frac{if_B f_{V_1} f_{V_2}}{X_0^{(\bar{B}V_1, V_2)}} \frac{m_B}{\lambda_B} \int_0^1 du dv \\
& \quad \left(\frac{\Phi_{\parallel}^{V_1}(u) \Phi_{\parallel}^{V_2}(v)}{\bar{u}\bar{v}} + r_{\chi}^{V_1} \frac{\Phi_{v_1}(u) \Phi_{\parallel}^{V_2}(v)}{\bar{u}v} \right), \tag{38}
\end{aligned}$$

for $i = 1 - 4, 9, 10$,

$$\begin{aligned}
H_i^0(V_1 V_2) &= -\frac{if_B f_{V_1} f_{V_2}}{X_0^{(\bar{B}V_1, V_2)}} \frac{m_B}{\lambda_B} \int_0^1 du dv \\
& \quad \left(\frac{\Phi_{\parallel}^{V_1}(u) \Phi_{\parallel}^{V_2}(v)}{\bar{u}v} + r_{\chi}^{V_1} \frac{\Phi_{v_1}(u) \Phi_{\parallel}^{V_2}(v)}{\bar{u}\bar{v}} \right), \tag{39}
\end{aligned}$$

for $i = 5, 7$, and $H_i^0(V_1 V_2) = 0$ for $i = 6, 8$. The transverse hard spectator terms $H_i^{\pm}(V_1 V_2)$ read

$$\begin{aligned}
H_i^-(V_1 V_2) &= \frac{2if_B f_{V_1}^{\perp} f_{V_2} m_{V_2}}{m_B X_-^{(\bar{B}V_1, V_2)}} \frac{m_B}{\lambda_B} \int_0^1 du dv \frac{\Phi_{\perp}^{V_1}(u) \Phi_{\perp}^{V_2}(v)}{\bar{u}^2 v}, \\
H_i^+(V_1 V_2) &= -\frac{2if_B f_{V_1} f_{V_2} m_{V_1} m_{V_2}}{m_B^2 X_+^{(\bar{B}V_1, V_2)}} \frac{m_B}{\lambda_B} \\
& \quad \int_0^1 du dv \frac{(\bar{u} - v) \Phi_{+}^{V_1}(u) \Phi_{+}^{V_2}(v)}{\bar{u}^2 \bar{v}^2}, \tag{40}
\end{aligned}$$

for $i = 1 - 4, 9, 10$,

$$\begin{aligned}
H_i^-(V_1 V_2) &= -\frac{2if_B f_{V_1}^{\perp} f_{V_2} m_{V_2}}{m_B X_-^{(\bar{B}V_1, V_2)}} \frac{m_B}{\lambda_B} \\
& \quad \int_0^1 du dv \frac{\Phi_{\perp}^{V_1}(u) \Phi_{+}^{V_2}(v)}{\bar{u}^2 \bar{v}}, \tag{41}
\end{aligned}$$

$$\begin{aligned}
H_i^+(V_1 V_2) &= -\frac{2if_B f_{V_1} f_{V_2} m_{V_1} m_{V_2}}{m_B^2 X_+^{(\bar{B}V_1, V_2)}} \frac{m_B}{\lambda_B} \\
& \quad \int_0^1 du dv \frac{(u - v) \Phi_{+}^{V_1}(u) \Phi_{-}^{V_2}(v)}{\bar{u}^2 v^2}, \tag{42}
\end{aligned}$$

for $i = 5, 7$, and

$$\begin{aligned}
H_i^-(V_1 V_2) &= -\frac{if_B f_{V_1} f_{V_2} m_{V_2}}{m_B X_-^{(\bar{B}V_1, V_2)}} \frac{m_B m_{V_1}}{m_{V_2}^2} \frac{m_B}{\lambda_B} \\
& \quad \int_0^1 du dv \frac{\Phi_{+}^{V_1}(u) \Phi_{\perp}^{V_2}(v)}{v \bar{u} \bar{v}}, \tag{43}
\end{aligned}$$

$$H_i^+(V_1 V_2) = 0, \tag{44}$$

for $i = 6, 8$. One can find that the expressions for $H_i^{\pm}(V_1 V_2)$ are independent of the choice for transverse polarization vectors.

The helicity dependent factorizable amplitudes defined by

$$X^{(\bar{B}V_1, V_2)} = \langle V_2(p_2, \epsilon_2^*) | J_{\mu} | 0 \rangle \langle V_1(p_1, \epsilon_1^*) | J^{\mu} | B \rangle \tag{45}$$

have the expressions

$$\begin{aligned}
X_0^{(\bar{B}V_1, V_2)} &= \frac{if_{V_2}}{2m_{V_1}} [(m_B^2 - m_{V_1}^2 - m_{V_2}^2)(m_B + m_{V_1}) \\
& \quad A_1^{BV_1}(q^2) - \frac{4m_B^2 P_c^2}{m_B + m_{V_1}} A_2^{BV_1}(q^2)], \tag{46}
\end{aligned}$$

$$\begin{aligned}
X_{\pm}^{(\bar{B}V_1, V_2)} &= -if_{V_2} m_B m_{V_2} \left[\left(1 + \frac{m_{V_1}}{m_B} \right) A_1^{BV_1}(q^2) \right. \\
& \quad \left. \mp \frac{2P_c}{m_B + m_{V_1}} V^{BV_1}(q^2) \right], \tag{47}
\end{aligned}$$

where $A_i^{BV_1}(i = 1, 2)$ and V^{BV_1} are weak form factors.

Penguin terms

At order α_s , corrections from penguin contractions are present only for $i = 4, 6$. For $i = 4$ we have [8,9]

$$\begin{aligned}
P_4^{h,p}(V_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left\{ c_1 [G_{V_2}^h(s_p) + g_{V_2}] \right. \\
& \quad + c_3 [G_{V_2}^h(s_s) + G_{V_2}^h(1) + 2g_{V_2}] \\
& \quad + (c_4 + c_6) \sum_{i=u}^b [G_{V_2}^h(s_i) + g'_{V_2}] \\
& \quad \left. - 2c_{8g}^{\text{eff}} G_g^h \right\} \tag{48}
\end{aligned}$$

where $s_i = m_i^2/m_b^2$ and the function $G_{V_2}^h(s)$ is given by

$$\begin{aligned}
G_{V_2}^h(s) &= 4 \int_0^1 du \Phi^{V_2, h}(u) \int_0^1 dx x \bar{x} \ln[s - \bar{u}x\bar{x} - i\epsilon], \\
g_{V_2} &= \left(\frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} \right) \int_0^1 \Phi^{V_2, h}(x) dx, \\
g'_{V_2} &= \frac{4}{3} \ln \frac{m_b}{\mu} \int_0^1 \Phi^{V_2, h}(x) dx, \tag{49}
\end{aligned}$$

with $\Phi^{V_2,0} = \Phi_{\parallel}^{V_2}$, $\Phi^{V_2,\pm} = \Phi_{\pm}^{V_2}$. For $i = 6$, the result for the penguin contribution is

$$P_6^{h,p}(V_2) = \frac{C_F \alpha_s}{4\pi N_c} \left\{ c_1 \hat{G}_{V_2}^h(s_p) + c_3 \left[\hat{G}_{V_2}^h(s_s) + \hat{G}_{V_2}^h(1) \right] + (c_4 + c_6) \sum_{i=u}^b \hat{G}_{V_2}^h(s_i) \right\}, \quad (50)$$

where the function $\hat{G}_{V_2}(s)$ is defined as

$$\begin{aligned} \hat{G}_{V_2}^0(s) &= 4 \int_0^1 du \Phi_{v_2}(u) \int_0^1 dx x \bar{x} \ln[s - \bar{u}x\bar{x} - i\epsilon], \\ \hat{G}_{V_2}^{\pm}(s) &= 0. \end{aligned} \quad (51)$$

The transverse penguin contractions vanish for $i = 6, 8$: $P_{6,8}^{\pm,p} = 0$. The $r_{\chi}^{M_2}$ term in Eq. (50) is factorized out so that when the vertex correction $V_{6,8}$ is neglected, a_6^0 contributes to the decay amplitude in the product $r_{\chi}^{V_2} a_6^0 \approx r_{\chi}^{V_2} P_6^0$ [8, 9]. For $i = 8, 10$,

$$P_8^{h,p}(V_2) = \frac{\alpha_{\text{em}}}{9\pi N_c} (c_1 + N_c c_2) \hat{G}_{V_2}^h(s_p), \quad (52)$$

$$P_{10}^{h,p}(V_2) = \frac{\alpha_{\text{em}}}{9\pi N_c} \{ (c_1 + N_c c_2) [G_{V_2}^h(s_p) + 2g_{V_2}] - 3c_{\gamma}^{\text{eff}} G_g^h \}. \quad (53)$$

For $i = 7, 9$,

$$\begin{aligned} P_{7,9}^{-,p}(V_2) &= -\frac{\alpha_{\text{em}}}{3\pi} C_{7\gamma}^{\text{eff}} \frac{m_B m_b}{m_{V_2}^2} + \frac{2\alpha_{\text{em}}}{27\pi} (c_1 + N_c c_2) \\ &\quad \left[\delta_{pc} \ln \frac{m_c^2}{\mu^2} + \delta_{pu} \ln \frac{\nu^2}{\mu^2} + 1 \right]. \end{aligned} \quad (54)$$

The relevant integrals for the dipole operators $O_{g,\gamma}$ are [8, 9]

$$\begin{aligned} G_g^0 &= \int_0^1 du \frac{\Phi_{\parallel}^{V_2}(u)}{\bar{u}}, \\ G_g^{\pm} &= 0. \end{aligned} \quad (55)$$

The dipole operators Q_{8g} and $Q_{7\gamma}$ do not contribute to the transverse penguin amplitudes at $\mathcal{O}(\alpha_s)$ due to angular momentum conservation [35].

Annihilation contributions

The annihilation contributions to the decay $\bar{B} \rightarrow V_1 V_2$ can be described in terms of $b_i^{p,h}$ and $b_{i,\text{EW}}^{p,h}$

$$\begin{aligned} &\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle V_1 V_2 | \mathcal{T}_B^{h,p} | \bar{B}^0 \rangle \\ &= i \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p f_B f_{V_1} f_{V_2} \sum_i (b_i^{p,h} + b_{i,\text{EW}}^{p,h}). \end{aligned} \quad (56)$$

The building blocks have the expressions

$$\begin{aligned} b_1 &= \frac{C_F}{N_c^2} c_1 A_1^i, & b_2 &= \frac{C_F}{N_c^2} c_2 A_1^i, \\ b_3 &= \frac{C_F}{N_c^2} [c_3 A_1^i + c_5 (A_3^i + A_3^f) + N_c c_6 A_3^f], \\ b_4 &= \frac{C_F}{N_c^2} [c_4 A_1^i + c_6 A_2^f], \\ b_{3,\text{EW}} &= \frac{C_F}{N_c^2} [c_9 A_1^i + c_7 (A_3^i + A_3^f) + N_c c_8 A_3^f], \\ b_{4,\text{EW}} &= \frac{C_F}{N_c^2} [c_{10} A_1^i + c_8 A_2^f], \end{aligned} \quad (57)$$

where we have omitted the superscripts p and h in above expressions for simplicity. The subscripts 1,2,3 of $A_n^{i,f}$ denote the annihilation amplitudes induced from $(V - A)(V - A)$, $(V - A)(V + A)$ and $(S - P)(S + P)$ operators, respectively, and the superscripts i and f refer to gluon emission from the initial and final-state quarks, respectively. V_1 contains an antiquark from the weak vertex and V_2 contains a quark from the weak vertex [8, 9]. The explicit expressions of weak annihilation amplitudes are:

$$\begin{aligned} A_1^{i,0}(V_1 V_2) &= \pi \alpha_s \int_0^1 du dv \\ &\quad \left\{ \Phi_{\parallel}^{V_1}(v) \Phi_{\parallel}^{V_2}(v) \left[\frac{1}{u(1-\bar{u}v)} + \frac{1}{u\bar{v}^2} \right] \right. \\ &\quad \left. - r_{\chi}^{V_1} r_{\chi}^{V_2} \Phi_{v_1}(u) \Phi_{v_2}(v) \frac{2}{u\bar{v}} \right\}, \end{aligned} \quad (58)$$

$$\begin{aligned} A_1^{i,-}(V_1 V_2) &= -\pi \alpha_s \frac{2m_{V_1} m_{V_2}}{m_B^2} \int_0^1 du dv \\ &\quad \left\{ \Phi_{-}^{V_1}(u) \Phi_{-}^{V_2}(v) \left[\frac{\bar{u} + \bar{v}}{u^2 \bar{v}^2} + \frac{1}{(1-\bar{u}v)^2} \right] \right\}, \end{aligned} \quad (59)$$

$$\begin{aligned} A_1^{i,+}(V_1 V_2) &= -\pi \alpha_s \frac{2m_{V_1} m_{V_2}}{m_B^2} \int_0^1 du dv \\ &\quad \left\{ \Phi_{+}^{V_1}(u) \Phi_{+}^{V_2}(v) \left[\frac{2}{u\bar{v}^3} - \frac{v}{(1-\bar{u}v)^2} - \frac{v}{\bar{v}^2(1-\bar{u}v)} \right] \right\}, \end{aligned} \quad (60)$$

$$\begin{aligned} A_2^{i,0}(V_1 V_2) &= \pi \alpha_s \int_0^1 du dv \left\{ \Phi_{\parallel}^{V_1}(v) \Phi_{\parallel}^{V_2}(v) \right. \\ &\quad \left[\frac{1}{\bar{v}(1-\bar{u}v)} + \frac{1}{u^2 \bar{v}} \right] \\ &\quad \left. - r_{\chi}^{V_1} r_{\chi}^{V_2} \Phi_{m_1}(u) \Phi_{m_2}(v) \frac{2}{u\bar{v}} \right\}, \end{aligned} \quad (61)$$

$$A_2^{i,-}(V_1 V_2) = -\pi\alpha_s \frac{2m_{V_1}m_{V_2}}{m_B^2} \int_0^1 du dv \left\{ \Phi_+^{V_1}(u) \Phi_+^{V_2}(v) \times \left[\frac{u+v}{u^2\bar{v}^2} + \frac{1}{(1-\bar{u}v)^2} \right] \right\}, \quad (62)$$

$$A_2^{i,+}(V_1 V_2) = -\pi\alpha_s \frac{2m_{V_1}m_{V_2}}{m_B^2} \int_0^1 du dv \left\{ \Phi_-^{V_1}(u) \Phi_-^{V_2}(v) \times \left[\frac{2}{u^3\bar{v}} - \frac{\bar{u}}{(1-\bar{u}v)^2} - \frac{\bar{u}}{u^2(1-\bar{u}v)} \right] \right\}, \quad (63)$$

$$A_3^{i,0}(V_1 V_2) = \pi\alpha_s \int_0^1 du dv \left\{ r_\chi^{V_1} \Phi_{m_1}(v) \Phi_{\parallel}^{V_2}(v) \frac{2\bar{u}}{u\bar{v}(1-\bar{u}v)} + r_\chi^{V_2} \Phi_{\parallel}^{V_1}(v) \Phi_{m_2}(v) \frac{2v}{u\bar{v}(1-\bar{u}v)} \right\}, \quad (64)$$

$$A_3^{i,-}(V_1 V_2) = -\pi\alpha_s \int_0^1 du dv \left\{ -\frac{m_{V_2}r_\chi^{V_1}}{m_{V_1}} \Phi_{\perp}^{V_1}(u) \Phi_-^{V_2}(v) \frac{2}{u\bar{v}(1-\bar{u}v)} + \frac{m_{V_1}r_\chi^{V_2}}{m_{V_2}} \Phi_+^{V_1}(u) \Phi_{\perp}^{V_2}(v) \frac{2}{u\bar{v}(1-\bar{u}v)} \right\}, \quad (65)$$

$$A_3^{f,0}(V_1 V_2) = \pi\alpha_s \int_0^1 du dv \left\{ r_\chi^{V_1} \Phi_{m_1}(u) \Phi_{\parallel}^{V_2}(v) \frac{2(1+\bar{v})}{u\bar{v}^2} - r_\chi^{V_2} \Phi_{\parallel}^{V_1}(u) \Phi_{m_2}(v) \frac{2(1+u)}{u^2\bar{v}} \right\}, \quad (66)$$

$$A_3^{f,-}(V_1 V_2) = -\pi\alpha_s \int_0^1 du dv \left\{ \frac{m_{V_2}r_\chi^{V_1}}{m_{V_1}} \Phi_{\perp}^{V_1}(u) \Phi_-^{V_2}(v) \frac{2}{u^2\bar{v}} + \frac{m_{V_1}r_\chi^{V_2}}{m_{V_2}} \Phi_+^{V_1}(u) \Phi_{\perp}^{V_2}(v) \frac{2}{u\bar{v}^2} \right\}, \quad (67)$$

and $A_1^{f,h} = A_2^{f,h} = A_3^{i,+} = A_3^{f,+} = 0$. V_1 contains an antiquark from the weak vertex with longitudinal fraction \bar{y} , while V_2 contains a quark from the weak vertex with momentum fraction x [8,9].

$A_{1,2}^{i,\pm}$ are suppressed by a factor of $m_1 m_2 / m_B^2$ relative to other terms, so only the annihilation contributions due to $A_3^{f,0}$, $A_3^{f,-}$, $A_{1,2,3}^{i,0}$ and $A_3^{i,-}$ are considered.

The logarithmic divergences in annihilation can be extract into unknown variable X_A

$$\int_0^1 \frac{du}{u} \rightarrow X_A, \quad \int_0^1 \frac{\ln u}{u} \rightarrow -\frac{1}{2}X_A. \quad (68)$$

3.3 The calculation of CP violation

In order to obtain the CP violation of $\bar{B} \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ in Eq.(16), we calculate the amplitudes t_ρ , t_ρ^a , t_ω , t_ω^a , p_ρ , p_ρ^a , p_ω and p_ω^a in Eqs.(18)(19) in the QCDF approach, which are tree-level and penguin-level amplitudes. The decay amplitudes for the process $\bar{B} \rightarrow \rho^0\rho^0(\omega)$ are in the QCD factorization as follows:

$$A_{\bar{B} \rightarrow \rho^0 \rho^0} = A_{\rho^0 \rho^0}(\alpha_4^p - \delta_{pu}\alpha_2 - \frac{1}{2}\alpha_{4,EW}^p - \frac{3}{2}\alpha_{3,EW}^p + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p + \delta_{pu}\beta_1 + 2\beta_4^p + \frac{1}{2}\beta_{4,EW}^p), \quad (69)$$

$$\begin{aligned} -2A_{\bar{B} \rightarrow \rho^0 \omega} &= A_{\rho^0 \omega}(\delta_{pu}\alpha_2 - \delta_{pu}\beta_1 + 2\alpha_3^p + \alpha_4^p \\ &+ \frac{1}{2}\alpha_{3,EW}^p - \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p \\ &- \frac{3}{2}\beta_{4,EW}^p) + A_{\omega \rho^0}(-\delta_{pu}\alpha_2 - \delta_{pu}\beta_1 + \alpha_4^p \\ &- \frac{3}{2}\alpha_{3,EW}^p - \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p \\ &- \frac{3}{2}\beta_{4,EW}^p), \end{aligned} \quad (70)$$

where

$$A_{V_1 V_2} = i \frac{G_F}{\sqrt{2}} \langle V_1 | (\bar{q}b)_{V-A} | B \rangle \langle V_2 | (\bar{q}q)_V | 0 \rangle \quad (71)$$

From Eq.(22), one can get

$$\alpha e^{i\delta_\alpha} = \frac{t_\omega + t_\omega^a}{t_\rho + t_\rho^a} = \frac{Q_2}{Q_1}, \quad (72)$$

where

$$\begin{aligned} Q_1 &= t_\rho + t_\rho^a \\ &= A_{\rho^0 \rho^0}[\alpha_4^{u,h} - \alpha_4^{c,h} - \delta_{pu}\alpha_2 \\ &- \frac{1}{2}(\alpha_{4,EW}^{u,h} - \alpha_{4,EW}^{c,h}) + \delta_{pu}\beta_1] \end{aligned} \quad (73)$$

$$\begin{aligned} Q_2 &= t_\omega + t_\omega^a \\ &= -\frac{1}{2}A_{\rho^0 \omega^0}[\delta_{pu}\alpha_2 - \delta_{pu}\beta_1 + \alpha_4^{u,h} \\ &- \alpha_4^{c,h} - \frac{1}{2}(\alpha_{4,EW}^{u,h} - \alpha_{4,EW}^{c,h}) \\ &- \frac{1}{2}A_{\omega^0 \rho^0}[-\delta_{pu}\alpha_2 - \delta_{pu}\beta_1 + \alpha_4^{u,h} \\ &- \alpha_4^{c,h} - \frac{1}{2}(\alpha_{4,EW}^{u,h} - \alpha_{4,EW}^{c,h})] \end{aligned} \quad (74)$$

In a similar way, with the aid of the Fierz identities, we can evaluate the penguin operator contributions p_ρ and p_ω . From Eq. (23) we have

$$\beta e^{i\delta_\beta} = \frac{p_\rho + p_\rho^a}{p_\omega + p_\omega^a} = \frac{Q_3}{Q_4}, \quad (75)$$

where

$$\begin{aligned} Q_3 &= p_\rho + p_\rho^a \\ &= A_{\rho^0\rho^0} \left[\left(-\frac{1}{2}\right)\alpha_{4,EW}^{c,h} - \left(\frac{3}{2}\right)\alpha_{3,EW}^{c,h} \right] \\ &\quad + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p + 2\beta_4^p + \frac{1}{2}\beta_{4,EW}^p \end{aligned} \quad (76)$$

$$\begin{aligned} Q_4 &= p_\omega + p_\omega^a \\ &= -\frac{1}{2}A_{\rho^0\omega} [2\alpha_3^c + \alpha_4^c] + \frac{1}{2}\alpha_{3,EW}^c \\ &\quad - \frac{1}{2}\alpha_{4,EW}^c + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p - \frac{3}{2}\beta_{4,EW}^p \\ &\quad - \frac{1}{2}A_{\omega\rho^0} (\alpha_4^c - \frac{3}{2}\alpha_{3,EW}^c - \frac{1}{2}\alpha_{4,EW}^c \\ &\quad + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p - \frac{3}{2}\beta_{4,EW}^p) \end{aligned} \quad (77)$$

Form Eq. (24) we have

$$r' e^{i(\delta_q + \phi)} = \frac{p_\omega + p_\omega^a}{t_\rho + t_\rho^a} = \frac{Q_4}{Q_1}, \quad (78)$$

$$r' e^{i\delta_q} = \frac{Q_4}{Q_1} \left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right|, \quad (79)$$

where

$$\left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right| = \frac{\sqrt{(1-\rho)^2 + \eta^2}}{(1 - \frac{\lambda^2}{2})(\sqrt{\rho^2 + \eta^2})}. \quad (80)$$

4 Input parameters

In the numerical calculations, we should input distribution amplitudes and the CKM matrix elements in the Wolfenstein parametrization. For the CKM matrix elements, which are determined from experiments, we use the results in Ref. [36]:

$$\begin{aligned} \bar{\rho} &= 0.132_{-0.014}^{+0.022}, & \bar{\eta} &= 0.341 \pm 0.013, \\ \lambda &= 0.2253 \pm 0.0007, & A &= 0.808_{-0.015}^{+0.022}, \end{aligned} \quad (81)$$

where

$$\bar{\rho} = \rho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}). \quad (82)$$

The general expressions of the helicity-dependent amplitudes can be simplified by considering the asymptotic distribution amplitudes for Φ_V, Φ_v :

$$\begin{aligned} \Phi_{\parallel}^V(u) &= 6u\bar{u}, & \Phi_v(u) &= 3(2u-1), \\ \Phi_{\perp}^V(u) &= 6u\bar{u}, & \Phi_+^V &= \int_u^1 dv \frac{\Phi_{\parallel}^V(v)}{v}, \\ \Phi_-^M &= \int_0^u dv \frac{\Phi_{\parallel}^V(v)}{v}. \end{aligned} \quad (83)$$

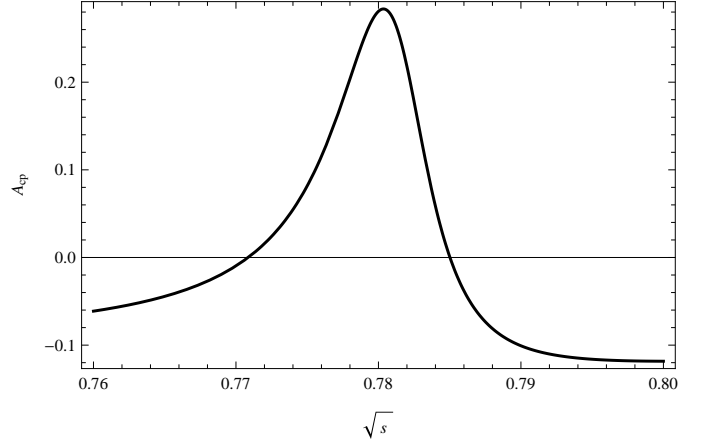


Fig. 1. Plot of A_{CP} as a function of \sqrt{s} corresponding to central parameter values of CKM matrix elements for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$.

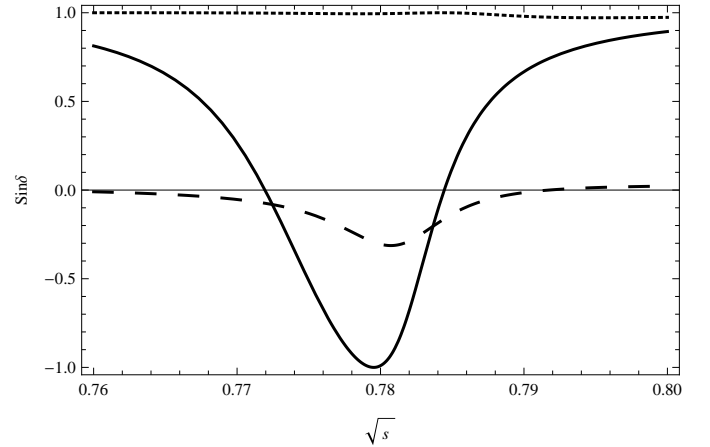


Fig. 2. Plot of $\sin \delta$ as a function of \sqrt{s} corresponding to central parameter values of CKM matrix elements for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. The solid (dashed and dotted) line corresponds to $\sin \delta_0$ ($\sin \delta_-$ and $\sin \delta_+$) respectively.

Power corrections in QCDF always involve endpoint divergences which produce some uncertainties. The endpoint divergence $X \equiv \int_0^1 dx/\bar{x}$ in the annihilation and hard spectator scattering diagrams is parameterized as

$$X_A = \ln \left(\frac{m_B}{\Lambda_h} \right) (1 + \rho_A e^{i\phi_A}), \quad (84)$$

$$X_H = \ln \left(\frac{m_B}{\Lambda_h} \right) (1 + \rho_H e^{i\phi_H}), \quad (85)$$

with the unknown real parameters $\rho_{A,H}$ and $\phi_{A,H}$ [8,9]. For simplicity, we shall assume that X_A^h and X_H^h are helicity independent: $X_A^- = X_A^+ = X_A^0$ and $X_H^- = X_H^+ = X_H^0$.

5 Numerical results

In the numerical results, we find that for the decay channel we are considering the CP violation can be enhanced

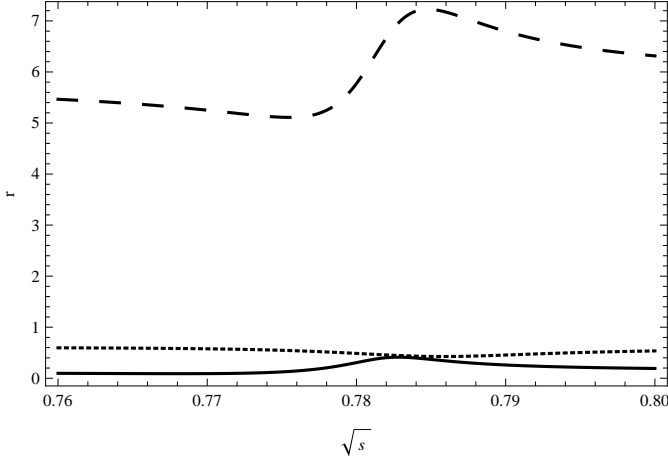


Fig. 3. Plot of r as a function of \sqrt{s} corresponding to central parameter values of CKM matrix elements for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. The solid (dashed and dotted) line corresponds to r_0 (r_- and r_+) respectively.

via $\rho - \omega$ mixing when the invariant mass of $\pi^+\pi^-$ is in the vicinity of the ω resonance. The uncertainties of the CKM matrix elements mainly come from ρ and η . In our numerical results, we let ρ and η vary between the limiting values. We find the results are not sensitive to the values of ρ and η . Hence, the numerical results are shown in Fig.1, Fig.2 and Fig.3 with the central parameter values of CKM matrix elements. From the numerical results, it is found that there is a maximum CP violating parameter value, A_{CP}^{max} , when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. In Fig.1, one can find that the maximum CP violating parameter reaches 28% in the case of $(\rho_{central}, \eta_{central})$.

From the Eq.(16) one can find that the CP violating parameter is related to $\sin \delta$ and r . In Fig.2, we show the plot of $\sin \delta_0$ ($\sin \delta_-$ and $\sin \delta_+$) as a function of \sqrt{s} . We can see that the $\rho - \omega$ mixing mechanism produces a large $\sin \delta_0$ ($\sin \delta_-$ and $\sin \delta_+$) at the ω resonance. As can be seen from Fig.2, the plots vary sharply in the cases of $\sin \delta_0$ and $\sin \delta_-$. Meanwhile, $\sin \delta_+$ changes weakly comparing with the $\sin \delta_0$ and $\sin \delta_-$. It can be seen from the Fig.3 that r_0 and r_- change more rapidly than r_+ when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance.

In the paper [23], we studied the enhanced CP violation for the decay channel $\bar{B}^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ in the naive factorization. Since non-factorizable contribution can not be calculated in the naive factorization, N_c was treated as an effective parameter. We found that the CP violating asymmetry was large and ranges from -82% to -98% via the $\rho - \omega$ mixing mechanism strongly depending on the value N_c when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance. However, the maximum CP violation only can reach 28% via double $\rho - \omega$ mixing in the QCD factorization. The naive factorization scheme has been shown to be the leading order result in the framework of QCD factorization when the radiative QCD corrections $O(\alpha_s(m_b))$ and the order $O(1/m_b)$ ef-

fects are neglected. The QCD factorization can evaluate systematically corrections to the results from the naive factorization. The distinction between the naive factorization and the QCDF mainly come from the strong phases of the QCD corrections. In the calculating process, we find that the annihilation contributions in QCDF which introduce the unknown parameters are small. Hence, the uncertainties of the results from the QCDF become small.

6 Summary and conclusions

In this paper, we studied the CP violation for the decay process $\bar{B}^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ due to the interference of $\rho - \omega$ mixing in the QCDF approach. This process induces two $\rho - \omega$ interference. It was found the CP violation can be enhanced at the region of $\rho - \omega$ resonance. As a result, the maximum CP violation could reach 28%. $\rho - \omega$ mixing is small due to the isospin violation. However, the mixing can produce a large strong phase, δ , in Eq. (21). This is because when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of ω , $s_\omega \sim im_\omega\Gamma_\omega$, and it becomes comparable with $\tilde{H}_{\rho\omega}$ in Eq. (21). In other words, $\rho - \omega$ mixing becomes important in the vicinity of ω . This is the reason why we can see large CP violation in the vicinity of ω . Beyond the $\rho - \omega$ interference region, the noticeable values of CP violation are caused by the strong phases provided by the Wilson coefficients.

The LHC experiments are designed with the center-of-mass energy 14 TeV and the luminosity $L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$. The heavy quark physics is one of the main topics of LHC experiments. Especially, LHCb detector is designed to make precise studies on CP asymmetries and rare decays of b-hadron systems. Recently, the LHCb Collaboration found clear evidence for direct CP violation in some three-body decay channels in charmless decays of B meson. Large CP violation is observed in $B^+ \rightarrow K^+K^-\pi^+$, $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$ in the region $m_{\pi^+\pi^-}^2 < 0.4 \text{ GeV}^2$ and $m_{\pi^+\pi^-}^2 > 15 \text{ GeV}^2$ [3]. LHCb experiment may collect data in the region of the invariant masses of $\pi^+\pi^-$ associated the ω resonance for detecting our prediction of CP violation.

In our calculations there are some uncertainties. The QCD factorization scheme provides a framework in which we can evaluate systematically corrections to the results obtained in the naive factorization scheme. However, when we take into account the nonfactorizable and chirally enhanced hard-scattering spectator and annihilation contributions which appear at order $O(\alpha_s(m_b))$ and $O(1/m_b)$, respectively, the involvement of the twist-3 hadronic distribution amplitudes leads to logarithmical divergence coming from the endpoint integrals. This brings large uncertainties in the predictions of the CP violating asymmetries in the QCD factorization scheme. Furthermore, in addition to the model dependence appearing in the factorized hadronic matrix elements just as in the naive factorization scheme, we cannot avoid the model dependence and process dependence of the hard-scattering spectator and annihilation contributions due to their dependence on the

hadronic distribution amplitudes and dependence on different processes. Such dependence will also appear if one tries to include other $1/m_b$ corrections and even higher order corrections. This leads to uncertain of our results.

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